

## Energy Balance of Controlled Thermonuclear Fusion

M. Hashmi and G. Staudenmaier  
Ruhr-Akademie der Wissenschaften  
Universitaet Center  
Postfach 25 05 20  
D-44801 Bochum, Germany

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### Abstract

It is shown that a discrepancy and incompatibility persist between basic physics and fusion-literature regarding the radiation losses from a thermonuclear plasma. Whereas the fusion-literature neglects the excitation or line radiation completely, according to basic physics it depends upon the prevailing conditions and cannot be neglected in general. Moreover, for a magnetized plasma, while the fusion-literature assumes a self-absorption or reabsorption of cyclotron or synchrotron radiation emitted by the electrons spiraling along the magnetic field, the basic physics does not allow any effective reabsorption of cyclotron or synchrotron radiation.

As is demonstrated, fallacious assumptions and notions, which somehow or other crept into the fusion-literature, are responsible for such a discrepancy.

In the present work, the theory is corrected. On the grounds of basic physics, a complete energy balance of magnetized and non-magnetized plasmas is presented for pulsed, stationary and self-sustaining operations by taking into account the energy release by reactions of light nuclei as well as different kinds of diffusive (conduction) and radiative (bremsstrahlung, cyclotron or synchrotron radiation and excitation radiation) energy losses. Already the energy losses by radiation make the energy balance negative. Hence, a fusion reactor — an energy producing device — seems to be beyond the realms of realization.

## 1. Introduction

The significance of energy loss by radiation from a thermonuclear plasma was recognized already at the early stage of fusion research [1-5]. However, although the basic physical processes that lead to emission, absorption and scattering of radiation or photon by bound and free electrons are all known, recognized and well established [6-17], regarding the radiation losses, there persists a discrepancy between basic physics [6-17] and its application to a fully ionized thermonuclear plasma [3-5, 18-33]. Whereas the basic processes conclusively yield the leakage of emitted radiation from a thermonuclear plasma, magnetized or non-magnetized, the fusion-literature neglects such an energy loss. This fundamental contradiction though has existed from the very beginning of fusion research but has remained unnoticed so far. The nature of this discrepancy primarily lies in the fact that fusion physics asserts, in particular, the reabsorption or self-absorption of the cyclotron or synchrotron radiation emitted by the free-spiraling charges of a magnetized plasma i.e. the radiation emitted by the helically moving charges in a magnetic field is reconverted by absorption into the kinetic energy of the emitting charges. Secondly, the excitation or line radiation, in general, is neglected completely. Only bremsstrahlung has been treated properly in fusion-literature.

In the present treatment it is demonstrated conclusively that, under the conditions prevailing in a magnetized thermonuclear plasma, the hypothesis of reabsorption or self-absorption of the cyclotron or synchrotron radiation is a severe flaw, since it is not compatible with basic physics. Hence, the hypothesis of reabsorption cannot be maintained. The reason is the erroneous application of physics to the magnetized thermonuclear plasma, which originates from a misinterpretation of fundamental laws. Moreover, it is shown that, in fact, the cyclotron or synchrotron radiation would completely leak out from a magnetized thermonuclear plasma. In addition, as far as the excitation or line radiation is concerned, it is found not to be negligible, in general, and would strongly depend upon the prevailing conditions.

Obviously, these radiation losses markedly influence the energy balance of a thermonuclear plasma.

The present paper describes a complete energy balance of a magnetized and non-magnetized plasma for pulsed, stationary and self-sustaining operations. It takes into

account the release of energy by fusion reactions as well as the different kinds of radiative (bremsstrahlung, cyclotron or synchrotron and excitation or line radiations) and diffusive (conduction) energy losses. Furthermore, a global energy balance which considers the efficiencies of converting the electrical energy into the plasma energy as well as the plasma energy into the electrical energy is presented.

The result of the present treatment is the fact that already the cyclotron radiation makes the energy balance of a magnetized thermonuclear plasma negative. This has far reaching consequences. Moreover, in case of a non-magnetized plasma i.e. in absence of cyclotron or synchrotron radiation, the excitation radiation is the dominating loss process; thereby making the energy balance negative. Further, the bremsstrahlung also yields a negative energy balance for the plasmas of higher atomic numbers ( $Z > 1$ ). Thus, the overall energy balance remains negative.

The probability of a binary interaction underlying a physical phenomenon is determined by the cross-section, which depends upon the energy. At a given energy, owing to different physical processes, different kinds of binary interactions could occur, each being given by a cross-section,  $[\sigma_{(a,b)}]_l$  ( $\text{cm}^2$ ); where  $l$  denotes a particular type of binary interaction between the particles  $a$  and  $b$ , each being an atom, electron, nucleus or photon. If  $n_a$  and  $n_b$  are the number of interacting particles per unit volume ( $\text{cm}^{-3}$ ) of species  $a$  and  $b$ , having a relative velocity  $v_{(a,b)}$  ( $\text{cm s}^{-1}$ ), the number of interactions of the kind  $l$  per unit volume per unit time ( $\text{cm}^{-3} \text{s}^{-1}$ ) is given by

$$\left(\frac{dn_l}{dt}\right) = n_a n_b [\sigma_{(a,b)}]_l v_{(a,b)} \quad (1)$$

If  $(\Delta E)_l$  (eV) is the energy transfer (release, loss or gain) per interaction of the kind  $l$ , the energy transfer rate per unit volume ( $\text{W cm}^{-3}$ ) is given by

$$P_l = 1.6 \times 10^{-19} n_a n_b [\sigma_{(a,b)}]_l v_{(a,b)} (\Delta E)_l \quad (2)$$

Experimentally, such a situation prevails in the laboratory when the energetic nuclei in a hot plasma, produced by ionizing the atoms, undergo non-threshold type nuclear reactions below the Coulomb barrier (tunnel effect), whose cross-sections as a function of

energy are given by the Gamov function [34-36]. The reaction cross-sections as a function of energy increase very rapidly. At a given energy, the reaction cross-sections also decrease very rapidly with increasing nuclear charges and reduced mass of the interacting nuclei.

Besides the nuclear reactions and other processes, a number of inevitable, inherent and intrinsic radiative processes e.g. excitation, recombination and bremsstrahlung occur, too. Moreover, if the charged particles describe curvatures, given by the radii of curvature, cyclotron or synchrotron radiation is emitted. This is the case in presence of an externally applied magnetic field.

Consequently, the ratio of the radiation loss rate per unit volume to energy release rate per unit volume, both given by Eq. (2), becomes a significant quantity. Only if this ratio is less than unity, an energy gain becomes possible; otherwise not.

The present paper investigates this very aspect of reactions by nuclei under different conditions.

## 2. Production of Nuclei and Associated Processes

Within a volume  $V$ , having a surface area  $A$ , energetic nuclei are produced by supplying energy to a gas or vapour, which is characterised by atomic density  $n_0$  ( $\text{cm}^{-3}$ ) and kinetic temperature  $T_0$  (eV). The mean velocity  $v_0$  of the atoms is related to the temperature  $T_0^*$  (K) by the relation

$$v_0 = \left[ \frac{8 k T_0^*}{\pi m} \right]^{1/2} \quad (3)$$

where  $k$  is the Boltzmann constant and  $m$  the atomic mass.  $kT_0^* = 1.6 \times 10^{-12} T_0$ . In case of a mixture of atoms of species p and q, characterised by atomic numbers  $Z_p$  and  $Z_q$  as well as atomic masses  $m_p$  and  $m_q$  respectively,

$$n_0 = (n_0)_p + (n_0)_q \quad (4)$$

Depending upon the conditions, the ionisation takes place by an energy transfer of  $(\Delta E)_{\text{ion}} = E_B + (1/2) m_e v_e^2$  per interaction to bound electrons of atoms from the free electrons,

nuclei or photons.  $E_B$ ,  $m_e$  and  $v_e$  are the binding energy, mass and velocity of the electron respectively. Hence, the ionisation rate and the required power  $P_{\text{ion}}$  for a given energy (or kinetic temperature) could be calculated by means of equations (1) and (2). However, simultaneously, excitation and recombination also occur. The cross-sections for these processes as a function of energy are given in the literature [37-39]. Still, it should be mentioned here that the excitation cross-sections (bound-bound transitions) are larger than the ionisation cross-sections (bound-free transitions), which are in turn much larger than the recombination cross-sections (free-bound transitions).  $(\Delta E)_{\text{exc}} = h\nu$  (eV) is the emitted energy quantum of frequency  $\nu$ ,  $h$  being the Planck's constant. Further,  $\Delta E^* \Delta \tau^* = \hbar$ , where  $\Delta E^*$  is the natural line-width and  $\Delta \tau^*$  the life-time of the emitted radiation, which is usually  $10^{-8}$  (s).  $(\Delta E)_{\text{rec}} = (1/2) m_e v_e^2 + h\nu$ . A detailed discussion is given later.

The free electrons of unit charge  $e$  and density  $n_e$  ( $\text{cm}^{-3}$ ), detached from the atoms, remain in the vicinity of nuclei of charges  $eZ_p$  and  $eZ_q$  and densities  $n_p$  and  $n_q$  ( $\text{cm}^{-3}$ ) such that

$$n_e = Z_p n_p + Z_q n_q \quad (\text{p} \neq \text{q}) \quad (5a)$$

or

$$n_e = Z n \quad (\text{p} = \text{q}) \quad (5b)$$

$n$  being the density of nuclei ( $\text{cm}^{-3}$ ) of charge  $eZ$ .

Due to the high mobility of electrons ( $m_e \ll m_p$  or  $m_q$ ), no space charge can build-up. Moreover, Coulomb or electrostatic forces will be effective only up to a distance of a few times the Debye length,  $\lambda_D = [(kT_e^*)/(4\pi e^2 n_e)]^{1/2} = 743 (T_e/n_e)^{1/2}$  (cm), beyond which the electrostatic field cannot penetrate and is shielded [40]. Further, the plasma frequency is given by  $\omega_p = v_e/\lambda_D$ . Electromagnetic wave of frequency  $\omega$  can propagate through the plasma only if  $\omega_p < \omega$ , and will otherwise be reflected [41]. Still, fluctuating electric fields could be present in such an assembly of particles [42,43]. Such a state of gas is nothing but a plasma.

A further energy transfer of  $\Delta E$  by Coulomb collisions among the charged particles as given by the collision cross-sections will not only cause an increase in the energy of the charged particles but also a change in their energy distributions — ultimately resulting in

Maxwellian velocity distributions owing to multiple collisions and the resulting scattering. Analogous to Eq. (3), the mean velocities of electrons and nuclei,  $v_e$ ,  $v_p$  and  $v_q$  are related to the temperatures  $T_e^*$ ,  $T_p^*$  and  $T_q^*$  (K) as well as the kinetic temperatures  $T_e$ ,  $T_p$  and  $T_q$  (eV) respectively. In case of equilibrium

$$T_e = T_p = T_q = T . \quad (6)$$

The values of collision cross-sections as well as the corresponding  $\Delta E$  and equipartitional times  $\tau_{eq}$  can be found in the literature [44-46].

With the production of energetic nuclei and electrons, bremsstrahlung (free-free transitions), electron-ion as well as electron-electron, is emitted, which will be discussed in detail later.

In addition, nuclear reactions set in. The reaction rate per unit volume is given by

$$\left( \frac{dn_{(r,s)}}{dt} \right) = n_p n_q [ \langle \sigma v \rangle_{(p,q)} ]_{(r,s)} , \quad (7)$$

where  $[ \langle \sigma v \rangle_{(p,q)} ]_{(r,s)}$  represents the product of the relative velocity of nuclei,  $v_{(p,q)}$ , and the reaction cross-section,  $\sigma_{(p,q)}$ , of the type (r,s) averaged over the Maxwellian velocity distribution corresponding to the kinetic temperature  $T$ . r and s denote the reaction products, usually two — either a nucleus and a neutron or both nuclei. The neutrons will leave the volume  $V$  instantaneously. Depending upon the mass deficit,  $\Delta m$ , a given amount of energy per reaction of the type (r,s),  $Q_{(r,s)}$  is released such that  $\Delta E = Q_{(r,s)} = \Delta m c^2$ ,  $c$  being the velocity of light.  $Q_{(r,s)}$  (eV) is shared by the reaction products  $Q_r$  and  $Q_s$  in form of their kinetic energies (inversely proportional to the masses  $m_r$  and  $m_s$ ) such that

$$Q_r = \left( \frac{m_s}{m_r + m_s} \right) Q_{(r,s)} \quad \text{and} \quad Q_s = \left( \frac{m_r}{m_r + m_s} \right) Q_{(r,s)} \quad (8)$$

The total energy release rate per unit volume by nuclear reactions,  $P_N$  ( $\text{W cm}^{-3}$ ), is given by

$$P_N = 1.6 \times 10^{-19} n_p n_q \sum_{(r,s)} [ \langle \sigma v \rangle_{(p,q)} ]_{(r,s)} Q_{(r,s)} \quad (9)$$

In case of identical nuclei ( $p = q$ ),  $n_p n_q = (1/2) n^2$ . Further  $n_p n_q = n_p^2$ , if  $p \neq q$  and  $n_p = n_q$ . This will be discussed in detail later.

Besides, the multiple electrostatic collisions among the particles contribute to the loss of particles (and the associated energy), too, from the volume  $V$  through the area  $A$ , by diffusion ( $\lambda \ll V^{1/3} \gg \lambda_D$ ), which is given by the diffusion coefficient  $D$ ;  $\lambda$  being the mean free path. The diffusion coefficients depend upon the prevailing conditions and can be found in the literature [47]. Usually, diffusion and radiation losses are so large that no appreciable density and kinetic temperature can build-up. Hence,  $P_N$  will remain very low. However, by applying an external magnetic field  $\mathbf{B}$  (gauss) which penetrates completely into the volume  $V$ , the diffusion coefficient across the magnetic field  $D_\perp$  is reduced markedly. This means that the magnetic pressure is much larger than the kinetic pressure i.e.

$$\beta_M = \frac{n_e T}{B^2/8\pi} \left[ 3 + \frac{n_r Q_r}{n_e T} + \frac{n_s Q_s}{n_e T} \right] < 1 \quad (10a)$$

where  $n_r$  and  $n_s$  are the densities of the reaction products  $r$  and  $s$ . Substituting  $M^* = n_r / n_e = n_s / n_e \ll 1$ , for charged reaction products, the Eq. (10a) transforms to

$$\beta_M = \frac{n_e T}{B^2/8\pi} \left[ 3 + 2M^* \frac{Q_{(r,s)}}{T} \right] < 1 \quad (10b)$$

and if one of the reaction products is a neutron ( $n_r/n_e = 0$ )

$$\beta_M = \frac{n_e T}{B^2/8\pi} \left[ 3 + M^* \frac{Q_s}{T} \right] < 1 \quad (10c)$$

Further, if different types of nuclear reactions take place simultaneously, the sum of the product of all the charged reaction products and their energies have to be taken. Yet, for a given  $T$ ,  $\beta_M$  and  $B$ , the maximum density  $(n_e)_{\max}$  and consequently the  $(P_N)_{\max}$  remain limited. The nuclei and electrons gyrate about and move along  $\mathbf{B}$ , describing helical motions (radii and pitch), whereby the directions of gyrations of nuclei and electrons oppose each other. Owing to their acceleration in curvatures, the gyrating charged particles generate the cyclotron or synchrotron radiation, whose intensity increases with decreasing  $\beta_M$ . This will be discussed later in detail.  $D_\perp$  prevailing under various conditions — for different

magnetic field geometries (homogeneous and non-homogeneous) as well as states (collisional and turbulent) — are given in the literature [42,47,48].

The characteristic life-time of charged particles  $\tau$ (s) can be defined by the relation

$$\tau = \left( \frac{r^2}{2D_{\perp}} \right) \quad (11)$$

$r$  being the radius perpendicular to  $\mathbf{B}$  and  $L$  the total length along  $\mathbf{B}$ , yielding  $V = \pi r^2 L$ .  $D_{\perp}$  is proportional to  $\beta_M$  and hence  $\tau$  is inversely proportional to  $\beta_M$ . Neglecting conduction, the particle and energy loss rates per unit volume related to diffusion,  $\Phi(\text{cm}^{-3} \text{s}^{-1})$  and  $P(\text{W cm}^{-3})$  respectively, are given by

$$\Phi = \left( \frac{n_e}{\tau} \right) \quad (12a)$$

$$P_p = \frac{3n_e T}{\tau} \quad (12b)$$

$n_0$ ,  $n_e$  and  $T$  as a function of time are given by the difference between the production (i.e. ionisation and increase in kinetic temperature) and loss (i.e. diffusion) rates; acquiring an equilibrium where the production and loss rates are equal. Initially, the degree of ionization is low,  $N^* = n_0/n_e \gg 1$ . The kinetic temperature is low, too. Production rate exceeds the loss rate. Gradually, the degree of ionisation and the kinetic temperature increase, still keeping the production rate larger than the loss rate. Later, the degree of ionisation achieves a high value,  $N^* = n_0/n_e \ll 1$ ; also the kinetic temperature. Finally, equilibrium is achieved. If the production is stopped now either by stopping the external energy supply or by eliminating the gas density  $n_0$  by some means, the density  $n_e$  will decay out exponentially with a time constant  $\tau$ . Only in the presence of a gas density  $n_0$ , the density  $n_e$  is sustained and a stationary state operation leading to saturation and equilibrium is achieved, where  $n_0$ ,  $n_e$ ,  $T$ ,  $n_r$ , and  $n_s$  remain constant.

In a self-sustaining system, the external energy supply is substituted by the kinetic energy of the charged reaction products.

It should perhaps be mentioned here that, in practice,  $n_e = 10^{15} \text{ (cm}^{-3}\text{)}$  cannot be achieved. For example, already at  $\beta_M = 10^{-1}$ ,  $B = 5 \times 10^4 \text{ (gauss)}$  and  $T > 10^4 \text{ (eV)}$ ,  $(n_e)_{\text{max}} < 2 \times 10^{14} \text{ (cm}^{-3}\text{)}$ . If  $\beta_M < 10^{-1}$ ,  $(n_e)_{\text{max}}$  would decrease further. At this density, the fre-

quencies of electron-electron and electron-nucleus collisions,  $\nu_{e-e}$  and  $\nu_{e-nuc}$  respectively, are smaller than  $2 \times 10^4$  ( $s^{-1}$ ) [44]. Further, in general,  $\tau > \tau_{eq}$ .

It is seen that, while producing energetic nuclei in order to achieve a given  $P_N$ , a number of basic physical processes lead to the production of photons or radiation as well — given by  $\sigma$  and  $\Delta E$ . Therefore, the total emitted power per unit volume  $P_{em}$  ( $W\ cm^{-3}$ ) is the sum of powers per unit volume emitted by excitation, recombination, bremsstrahlung and cyclotron or synchrotron radiation processes i.e.  $P_{em} = P_{exc} + P_b + P_{cyc} + P_{rec}$ .

### 3. Evaluation of Power by Nuclear Reactions, $P_N$

The values of  $Q_{(r,s)}$ ,  $[\sigma_{(p,q)}]_{(r,s)}$  and  $[\langle \sigma v \rangle_{(p,q)}]_{(r,s)}$  as a function of energy for different nuclear reactions are obtained from the literature [49-51]. The values of  $\Sigma [\langle \sigma v \rangle_{(p,q)}]_{(r,s)}$   $Q_{(r,s)}$  ( $eV\ cm^3\ s^{-1}$ ) for a number of reactions by light nuclei are given in Table I. Hence,  $P_N$  for any density and kinetic temperature can be obtained by Eq. (9).

### 4. Evaluation of Total Emitted Power, $P_{em}$

#### 4.1. Excitation Radiation

The energy loss rate per unit volume,  $P_{exc}$  ( $W\ cm^{-3}$ ), is given by

$$P_{exc} = 1.6 \times 10^{-19} n_0 n_e \Sigma [\langle \sigma v_e \rangle_{exc}]_v h \nu \quad (13)$$

Since  $v_e \gg v_p$  or  $v_q$ , the terms due to nuclei are neglected.  $[\langle \sigma v_e \rangle_{exc}]_v$ , corresponds to the mean value of the product of the relative velocity of the electron,  $v_e$ , and the electron-atom collision cross-section for the energy transition  $(\Delta E)_{exc} = h \nu < E_B = e V_{ion}$ ;  $V_{ion}$  being the ionisation potential of the atom. The transition  $h \nu = e V_{ion}$  leads to continuum and is given by the ionisation cross-section. The sum includes all the excited levels from the ground state. It can be replaced by  $[(\sigma v_e)_{exc}]_{total} \langle hv \rangle$ , which represents now the product

of total excitation cross-section and the mean energy transition. Equation (13) can be denoted now in a simple form by

$$P_{\text{exc}} = 1.6 \times 10^{-19} n_0 n_e [(\sigma v_e)_{\text{exc}}]_{\text{total}} \langle hv \rangle \quad (14)$$

Substituting the values of  $n_e$  and  $n_0$  from Eqs. (5a) and (5b) in Eq. (14),

$$P_{\text{exc}} = 1.6 \times 10^{-19} N^* n^2 Z^2 [(\sigma v_e)_{\text{exc}}]_{\text{total}} \langle hv \rangle \quad (15a)$$

for  $p = q$ .

$$P_{\text{exc}} = 1.6 \times 10^{-19} N^* n_p^2 (Z_p + Z_q)^2 [(\sigma v_e)_{\text{exc}}]_{\text{total}} \langle hv \rangle \quad (15b)$$

for  $p \neq q$  and  $n_p = n_q$ .

The value of  $P_{\text{exc}}$  can be best demonstrated for the isotopes of hydrogen ( $Z = 1$ ), which not only possess the minimum value of  $P_{\text{exc}}$  but also the maximum value of  $P_N$ . As  $Z$  increases,  $\sigma_{\text{exc}}$  increases, too, but  $P_N$  decreases. Above  $10^3$  (eV),  $\sigma_{\text{ion}}$  and  $\sigma_{\text{exc}}$  decrease with increasing energy. The first excited energy level of hydrogen atoms is at 10.2 (eV), the next being at 12.1 (eV), etc.;  $(e V_{\text{ion}})_{\text{hydrogen}} = 13.6$  eV. The mean value of  $[(\sigma v_e)_{\text{exc}}]_{\text{total}} \langle hv \rangle$  is assessed to be approximately  $10^{-7}$  (eV cm<sup>3</sup> s<sup>-1</sup>) for plasma temperatures between  $10^4$  and  $10^5$  (eV) [52].

#### 4.2. Recombination Radiation

The energy loss rate per unit volume,  $P_{\text{rec}}$  (W cm<sup>-3</sup>), is given by

$$P_{\text{rec}} = n_e(n_q + n_p) \langle \sigma v_e \rangle_{\text{rec}} (\Delta E)_{\text{rec}} \quad (16)$$

In case of identical nuclei,  $n_p + n_q = n$ . The cross-sections as a function of energy can be found in the literature [53]. However, they are already very small even at very low temperatures, e.g.  $10^{-21}$  (cm<sup>2</sup>) at 1 (eV) and decrease with increasing temperature.  $P_{\text{rec}}$  will play a role only if  $n_e \geq 10^{19}$  (cm<sup>-3</sup>). For the present consideration, it is neglected.

### 4.3. Bremsstrahlung

A continuous frequency-spectrum of electron-nucleus bremsstrahlung is emitted when the free electron is deflected and accelerated by the electric field of the nucleus of charge  $eZ$  and mass  $m$ . The energy is radiated only during the time of interaction, which is very short and extends over the distance of actual acceleration only [54-56]. In the same way electron-electron bremsstrahlung can occur, too, but is negligible in the present case [57]. However, this is neglected. The contribution of the charged reaction products can also be neglected, since  $M^* \ll 1$ . The power radiated per unit volume,  $P_b$  ( $\text{W cm}^{-3}$ ), for identical nuclei ( $p = q$ ) is given by

$$P_b = n_e n v_e (\sigma_{\text{electron-nucleus}})_b (\Delta E)_b \quad (17)$$

In Eq. (17),  $(\langle \sigma v_e \rangle_{\text{electron-nucleus}})_b (\Delta E)_b$  has been replaced by  $v_e (\sigma_{\text{electron-nucleus}})_b (\Delta E)_b$ , because, for non-relativistic energies, the product  $(\sigma_{\text{electron-nucleus}})_b (\Delta E)_b$  is independent of the kinetic temperature and given by [58].

$$(\sigma_{\text{electron-nucleus}})_b (\Delta E)_b = \left( \frac{16e^2}{3\hbar c^2} \left( \frac{e^2}{m_e c^2} \right)^2 \right) m_e c^2 Z^2 \quad (18)$$

For relativistic energies, it is also given in the literature. Now, by combining Eqs. (17), (18), (5a) and (5b),  $P_b$  for non-relativistic energies can be expressed by

$$P_b = 1.69 \times 10^{-32} n^2 Z^3 T_e^{1/2} \quad (\text{W cm}^{-3}) \quad (19a)$$

for  $p = q$  and

$$P_b = 1.69 \times 10^{-32} n_p^2 (Z_p + Z_q)(Z_p^2 + Z_q^2) T_e^{1/2} \quad (\text{W cm}^{-3}) \quad (19a)$$

for  $p \neq q$  and  $n_p = n_q$ .

For the particle energies considered here, the emitted spectra are in the ultraviolet and soft x-ray regions.

### 4.4. Cyclotron or Synchrotron Radiation

Contrary to quantum mechanical processes discussed so far e.g. bremsstrahlung, excitation and recombination radiations, the process of emission of cyclotron or synchrotron radiation is classical [9,16]. It is not a transition. The emitted radiation is not quantized. The quantum

mechanical effects will be effective for electron energies of about  $10^{16}$  (eV) where  $h\nu / c \approx m_e c$ . The helical motion of electrons along the magnetic field  $\mathbf{B}$  (gauss) results in the continuous emission of radiation owing to the continuous and high acceleration of charges. Only a very small fraction  $\delta E$  of the total energy of the electron  $E$  is radiated per revolution,

$$\delta E = \left( \frac{4\pi}{3} \frac{e^2}{R} \right) \beta^3 \gamma^4 \quad (20)$$

where the radius of gyration  $R = (m_e v_{e\perp} c) / eB$ ,  $\gamma = E / m_e c^2 = (1 - \beta^2)^{-1/2}$  and  $\beta = v_e / c$ . The radiation emitted by nuclei is negligible, since  $m_e \ll m_p$  or  $m_q$ . The radiation is plane polarized in the plane of acceleration. In relativistic case, the angular distribution is peaked in the direction of initial velocity vector, while in non relativistic case,  $\sin^2 \theta$  dependence exists,  $\theta$  being the angle between the acceleration and direction of emission [59,70]. The emission of radiation takes place at every point of the curvature described by the electron. The intensity is uniformly distributed over the whole plane of gyration. At any point of observation, it just appears as a pulse of very short duration. For an electron moving in a circular orbit, the pulse appears repeatedly at the angular frequency  $\omega_0 = v_{e\perp} / R = eB / m_e c$ . The Fourier spectrum of the pulse will contain frequencies that are integral multiples of  $\omega_0$ , having an upper limit approximately equal to the reciprocal of the duration of the pulse. For non-relativistic energies, the spectrum will contain a discrete set of frequencies, multiples of  $\omega_0$ ,  $\omega_0$  having more than 90 % of the total intensity, while for relativistic energies, the emitted spectrum will be a broad one. The details are available in the literature [9,16].

The emitted power per unit volume is given by

$$P_c = n_e \left( \frac{\omega_0}{2\pi} \right) \delta E = \left( \frac{2}{3} \right) \omega_0 \left( \frac{e^2}{R} \right) \beta^3 \gamma^4 n_e \quad (21a)$$

It can be transformed for  $\gamma = 1$  by Eq. (10a) neglecting the nuclear reaction products into the form

$$P_c = 32\pi \frac{e^4}{m_e^3 c^5} = n_e^2 T_e^2 \beta_M^{-1} \quad (21b)$$

$$= 7.5 \times 10^{-38} n_e^2 T_e^2 \beta_M^{-1} \quad (\text{W cm}^{-3}) \quad (21c)$$

Equation (21c) can be further transformed by the help of Eqs. (5a) and (5b) into the form

$$P_c = 7.5 \times 10^{-38} n^2 Z^2 T_e^2 \beta_M^{-1} \quad (\text{W cm}^{-3}) \quad (21d)$$

for  $p = q$  and

$$P_c = 7.5 \times 10^{-38} n_p^2 (Z_p + Z_q)^2 T_e^2 \beta_M^{-1} \quad (\text{W cm}^{-3}) \quad (21e)$$

for  $p \neq q$  and  $n_p = n_q$ .

For electron energies between  $10^3$  (eV) and  $10^6$  (eV) and a magnetic field up to  $10^5$  (gauss), the emitted radiation is in the infrared and microwave regions. Further, the electrons do not gyrate in phase and the emitted radiation is incoherent.

### 5. Evaluation of Radiated Power, $P_{\text{rad}}$

$P_{\text{em}}$  consists of different intensities and frequencies [ $\nu_b \leq 10^{18}$  ( $\text{s}^{-1}$ ),  $\nu_{\text{exc.}} \leq 10^{16}$  ( $\text{s}^{-1}$ ) and  $\nu_c \geq 10^{11}$  ( $\text{s}^{-1}$ )]. In order to calculate now the radiated power per unit volume that will leak out from the plasma,  $P_{\text{rad}}$  ( $\text{W cm}^{-3}$ ), one has to investigate whether or not the emitted radiation will be reabsorbed by the plasma i.e. the energy of radiation will be reconverted into the kinetic energy of free-spiraling charges. This can be evaluated on the basis of interactions of photon or radiation with matter and their cross-sections which are all well known [10-12]. The basic processes are:

#### A. Quantum mechanical

1. Photo-excitation [ $\Delta E = h \nu < E_B$ ]
2. Photo—ionisation [ $\Delta E = h \nu = E_B + (\frac{1}{2}) m_e v_e^2$ ]
3. Compton effect [ $\Delta E = h \nu'$ ,  $\nu' < \nu$ , if  $h \nu/c > m_e c$ ]
4. Pair production [ $\Delta E = h \nu$ , if  $h \nu \geq 2 m_e c^2$ ]
5. Photonuclear interactions

The emitted radiation-spectra are such that pair production as well as well as photo-nuclear interactions are ruled out. The interactions of radiation or photon with bound electrons,

namely, photoionisation (photoelectric effect) and photoexcitation would occur only in presence of atoms (see section 2). In any case, these would not contribute to the energy transfer to the free-spiraling charges of the plasma.

Inelastic scattering by free electrons as given by Compton effect converts the energy of radiation or photon into the kinetic energy of electrons only partially [60,61]. In the present case, the energies of excitation radiation and cyclotron or synchrotron radiation are such that Compton effect is ruled out. For bremsstrahlung, too, it is negligible because of low frequencies and intensities of photons, low electron densities as well as small cross-sections.

## B. Classical

### 1. Scattering by Free Electrons [ $(h\nu/c) < m_e c$ ] and Collisional Damping

Under the influence of the oscillating electric field of the incident radiation or photon, a free electron oscillates with the incident frequency  $\nu$  about its mean position. This gives rise to the emission of a secondary wave of the same frequency  $\nu$ . This means that the incident radiation or photon is scattered elastically with the well known Thomson scattering cross-section [11].

If the energy of the electron oscillating with frequency  $\nu$  is transferred to the neighbouring electrons or nuclei by collisions, damping will occur. Hence, the dissipation of radiation energy in the plasma would be observed. The extent of damping is determined by the lossiness of the plasma given by  $\sigma'/\omega\varepsilon$ , where  $\sigma'$  is the electrical conductivity for the radiation frequency  $\omega$  and  $\varepsilon$  the dielectric constant [62-64,73]. For  $\omega \gg \nu_{e-e}$  or  $\nu_{e-nuc.}$ ,  $\sigma'$  is proportional to the electron-nucleus,  $\nu_{e-nuc.}$ , and electron-electron,  $\nu_{e-e}$ , collision frequencies. Since  $\nu_{e-e} \approx \nu_{e-nuc.} < 10^4$  [s<sup>-1</sup>] [44,65] is much less compared to  $\nu_c > 10^{11}$  [s<sup>-1</sup>] for the conditions of thermonuclear plasma, the resulting e-folding length for energy dissipation by collisions is of the order of  $10^6$  [cm].

### 2. Collisionless or Resonance Absorption

The process to be considered next is the so-called collisionless or resonance absorption of electromagnetic waves. While in a non-magnetized plasma the plasma frequency,  $\omega_p$ , determines the dispersion relation, in a magnetized plasma there will be two cut-offs and two

resonances [66-68]; but only the resonance at the electron cyclotron frequency,  $\omega_c$ , is of interest here. Electrons spiraling along the magnetic field can be accelerated by the electric field vector of the right hand circularly polarized wave rotating in phase with the motion of electrons [69]. The question is now whether or not the emitted cyclotron radiation fulfills these conditions of resonance? The emitted power per unit solid angle is proportional to  $\sin^2\theta$  in the non-relativistic case; where  $\theta$  is the angle between the acceleration and direction of propagation, the electric vector being in the plane of acceleration [59,70]. Thus, the conditions of resonance would be fulfilled only for the very small fraction of radiation, propagating parallel or nearly parallel to the direction of the magnetic field. Further, keeping in mind that neither the spiraling electrons nor the emitted radiation are in phase, the process of resonance absorption becomes even less probable. The same is even more true for relativistic case. In addition, in a toroidal geometry, the zone of resonance is reduced further due to the inhomogeneity of the magnetic field in the direction of major radius.

In summary, it can be stated now that there is no physical process that would lead to a noticeable reabsorption or self-absorption of emitted radiation from a thermonuclear plasma.

Nevertheless, in order to convert the cyclotron or synchrotron radiation into the kinetic energy of electrons, a very diverse proposal for the re-absorption has been forwarded [3,4]. The magnetic fusion rests solely upon this proposal [5,18-33]. It applies erroneously the black body radiation and the related Kirchhoff's law to calculate the re-absorption coefficient of the cyclotron or synchrotron radiation by the free-spiraling electrons of the plasma. This is actually a flaw in many ways. In order to demonstrate it conclusively, the salient features of black body radiation have to be recalled first [6,8,13-15].

Black body radiation is observed in a cavity under very special conditions. The cavity — a sort of small enclosure for radiation — is surrounded by solid material walls consisting of atoms. The radiation that fills the cavity, called the black body radiation, is obtained by heating the material below the melting point. The radiation from such a body depends only on the temperature  $T_1$  [K] to which it is raised and not at all on the material of the body. In other words, the interchange of radiation energy between the cavity and the surrounding material forming the cavity will lead to the establishment and maintenance of a state of equilibrium completely defined by the temperature. In this state of equilibrium, the encl-

sed space, the cavity, will be filled with radiation whose spectral distribution and total intensity are functions exclusively of the temperature  $T_1$  and given by well known Planck's distribution function,

$$I_\nu d\nu = \left( \frac{2 h \nu^3}{c^2 (e^{h\nu/kT} - 1)} \right) d\nu$$

The integration yields,

$$\int I_\nu d\nu = \left( \frac{2\pi^5 k^4}{15c^2 h^3} \right) T_1^4 = \sigma^* T_1^4 \quad ,$$

the Stephan-Boltzmann radiation law.  $I_\nu$  is the spectral energy flux density and  $\sigma^*$  is a constant. Kirchoff's law relates emission coefficient,  $E_\nu^{**}$  ( $\text{erg cm}^{-3} \text{ s}^{-1}$ ), to absorption coefficient,  $\alpha_\nu^{**}$  ( $\text{cm}^{-1}$ ), such that  $E_\nu^{**}/\alpha_\nu^{**}=I_\nu$  for each value of  $\nu$  over the whole spectrum of black body. Since there exists a thermal equilibrium between the radiation from the walls to cavity and vice versa, the emission energy flux density from the walls to cavity is equal to the incoming energy flux density from the cavity to the walls, both given by  $I_\nu$  ( $\text{erg cm}^{-2} \text{ s}^{-1}$ ), for each value of  $\nu$  over the whole spectrum. Moreover, the total incoming energy flux density from the cavity to the material, consisting of different frequencies, is absorbed completely inside the material i.e. the material is optically thick, namely  $\alpha_\nu^{**}$  ( $\text{cm}^{-1}$ ) multiplied by the thickness of the material ( $\text{cm}$ )  $\gg 1$  (=1 yields the e-folding length) for all values of  $\nu$  over the whole spectrum. Since the matter is optically thick, also  $E_\nu^{**}$  ( $\text{erg cm}^{-3} \text{ s}^{-1}$ ) multiplied by the thickness of the material ( $\text{cm}$ )  $\gg 1$ , showing that the emission energy flux density as given by  $I_\nu$  for all the values of  $\nu$  over the whole spectrum originates from the surface of the walls. Planck's explanation of black body radiation was the origin of quantum theory. Later, Einstein introduced spontaneous and induced emission coefficients of the quantized states of excited atoms, which is nothing but thermal or electron and photo excitations of atoms respectively in the present-day terminology [8]. Hence, the outcome of the black body radiation is the thermal equilibrium between excitation and de-excitation of atoms and molecules, leading to  $I_\nu$ , given by excitation or line radiation, namely, quantized bound-bound transitions below the binding energies, where the radiation is first absorbed and then re-emitted such that  $E_{mn} = h (\nu_m - \nu_n) = h (\nu_n - \nu_m) = E_{nm}$ , having  $\Delta E^* \Delta\tau^* = \hbar$ ;  $\Delta\tau^* \approx 10^{-8}$  (s); where the symbols have their usual meaning

[6-8,11,13,15,17,71]. Consequently, the related Kirchhoff's law represents in thermal equilibrium the ratio of de-excitation coefficient ( $\text{erg cm}^{-2} \text{s}^{-1}$ ) to excitation coefficient ( $\text{cm}^{-1}$ ) of atoms, given by  $I_\nu$ , for all values of  $\nu$  over the whole spectrum [6-8,11,13,15,17,71].

However, by increasing the temperature  $T_1$  of a material body continuously also a change in its state occurs — leading to gaseous state — and further the gas is ionized by collisional processes forming a plasma, consisting of free electrons and nuclei, as already described.

Now, the reason for the flaw is evident. It is due to the fallacious application of radiation laws, first introduced by Trubnikov [3,4], subsequently adopted by other authors [5,18-33,72], as follows:

(1) A self-absorption or reabsorption of the cyclotron or synchrotron radiation by the emitting electrons is supposed without giving any physical absorption mechanism i.e. showing that such a physical process really exists. Hence, the absorption coefficient remains unknown.

(2) It is assumed further, if the plasma becomes optically thick, it will radiate like a black body, but the black body conditions are never attained. This is a superfluous and irrelevant supposition. In order to determine the optical thickness, the absorption coefficient is required. But the excitation and de-excitation of atoms (black body radiation) do not contribute to the reabsorption of the cyclotron or synchrotron radiation by free-spiraling electrons as has been shown in the present treatment.

(3) The absorption coefficient of cyclotron radiation by free electrons is calculated with the help of Rayleigh-Jeans intensity,  $I_{R-J}$ , and  $E_{\nu}^{**} = P_c$  by the relation  $(E_{\nu}^{**}/\alpha_{\nu}^{**})_{T=\text{constant}} = I_{R-J} = \omega_c^2 T_c / 8\pi^2 c^2$  which should according to Trubnikov [3,4] and other authors [5,18-33,72] represent the Kirchhoff's law. This procedure is incorrect for several reasons. Firstly, since  $E_{\nu}^{**}$  and  $I_\nu$  have been replaced by  $P_c$  and  $I_{R-J}$  respectively, the relation is not the Kirchhoff's law. Secondly, neither the present relation nor the Kirchhoff's law can be utilized to determine the absorption coefficient of cyclotron or synchrotron radiation by free-spiraling electrons as has been shown in the present treatment.

Thus, it is seen that the results presented by Trubnikov and subsequently adopted by other authors have no physical meaning. Now, it has been conclusively demonstrated that

the emitted radiation from the thermonuclear plasma will leak out almost completely. Finally, it is seen that  $P_{em} \approx P_{rad}$ . The energy balance of the plasma is given by

$$\left(\frac{P_P}{P_N}\right) + \left(\frac{P_{rad}}{P_N}\right) < 1 \quad (22)$$

Therefore,  $P_{rad}/P_N$  has to be evaluated first. Only if  $P_{rad}/P_N < 1$ ,  $P_P/P_N$  has to be investigated further.

### 6. The Ratio $P_{rad}/P_N$

The total radiated power,  $P_{rad}$  ( $\text{W cm}^{-3}$ ), is given by

$$P_{rad} = P_b + P_c + P_{exc} + P_{rec} \quad (23)$$

Thus, the ratio  $P_{rad}/P_N$  can be estimated.

(1) The ratio  $P_b/P_N$  is given by Eqs. (19a), (19b) and (9)

$$\left(\frac{P_b}{P_N}\right) = 2.12 \times 10^{-13} \left( \frac{Z^3 T_e^{1/2}}{\sum_{(r,s)} [\langle \sigma v \rangle_{(pq)}]_{(r,s)} Q_{(r,s)}} \right) \quad (24a)$$

for  $p = q$  and

$$\left(\frac{P_b}{P_N}\right) = 1.06 \times 10^{-13} \left( \frac{(Z_p + Z_q)(Z_p^2 + Z_q^2) T_e^{1/2}}{\sum_{(r,s)} [\langle \sigma v \rangle_{(pq)}]_{(r,s)} Q_{(r,s)}} \right) \quad (24b)$$

for  $p \neq q$  and  $n_p = n_q$ .

The ratio  $P_b/P_N$  for a number of reactions by light nuclei at different kinetic temperatures is shown in Table II. Wherever the ratio is larger than unity, already the bremsstrahlung is sufficient to make the energy balance negative. The functional dependences of  $P_b$  and  $P_N$  are such that for a given reaction, the ratio  $P_b/P_N$  decreases as  $T$  increases. On the contrary, at a given temperature, the ratio increases with increasing  $Z$ . It is seen that for  $Z_p = 1$  and  $Z_q > 3$ , the ratio becomes larger than unity at any temperature. For a self-sustaining system of reaction no. 2, it can be obtained by multiplying the values as given in Table II by

a factor of 5, since the energies of helium nucleus and neutron per reaction are  $3.5 \times 10^6$  and  $14.1 \times 10^6$  (eV) respectively.

(2) The ratio  $P_c/P_N$  is given by Eqs. (21d), (21e) and (9)

$$\left(\frac{P_c}{P_N}\right) = 9.4 \times 10^{-19} \left( \frac{Z^2 T_e^2 \beta_M^{-1}}{\sum_{(r,s)} [\langle \sigma v \rangle_{(pq)}]_{(r,s)} Q_{(r,s)}} \right) \quad (25a)$$

for  $p = q$  and

$$\left(\frac{P_c}{P_N}\right) = 4.7 \times 10^{-19} \left( \frac{(Z_p + Z_q)^2 T_e^2 \beta_M^{-1}}{\sum_{(r,s)} [\langle \sigma v \rangle_{(pq)}]_{(r,s)} Q_{(r,s)}} \right) \quad (25b)$$

for  $p \neq q$  and  $n_p = n_q$ .

The ratio  $P_c/P_N$  is shown in Table III for a number of reactions by light nuclei at different kinetic temperatures and  $\beta_M = 10^{-1}$ . It is seen that for  $\beta_M = 10^{-1}$ , the ratio  $P_c/P_N \geq 1$ . This critical value is achieved for a self-sustaining system of reaction no. 2 already at  $\beta_M = 5 \times 10^{-1}$ . In practice, however,  $\beta_M < 10^{-1}$  because  $\tau$  increases as  $\beta_M$  decreases. Thus,  $\beta_M$  influences  $N^*$  e.g.  $N^*$  increases as  $\beta_M$  increases. This is discussed in detail later.

(3) The ratio  $P_{exc}/P_N$  is given by Eqs. (15a), (15b) and (9)

$$\left(\frac{P_{exc}}{P_N}\right) = \left( \frac{2N^* Z^2 [(\sigma v_e)_{exc}]_{total} \langle hv \rangle}{\sum_{(r,s)} [\langle \sigma v \rangle_{(pq)}]_{(r,s)} Q_{(r,s)}} \right) \quad (26a)$$

for  $p = q$  and

$$\left(\frac{P_{exc}}{P_N}\right) = \left( \frac{N^* (Z_p + Z_q)^2 [(\sigma v_e)_{exc}]_{total} \langle hv \rangle}{\sum_{(r,s)} [\langle \sigma v \rangle_{(pq)}]_{(r,s)} Q_{(r,s)}} \right) \quad (26a)$$

for  $p \neq q$  and  $n_p = n_q$ .

The ratio  $P_{exc}/P_N$  is the least for  $Z=1$  and  $M^* \ll 1$ . At various kinetic temperatures  $N^*$  is given by the ratio of the production rate to loss rate, which are equal in saturation or equilibrium, yielding the minimum value. Any increase in  $M^*$  will correspond to a further increase in  $N^*$ . However, the numerical values of  $N^*$  will depend upon the conditions prevailing actually and can be determined only empirically from case to case.

Since the other factors are known, the critical values of  $N^*$  for  $Z = 1$ ,  $N^*_{\text{critical}}$ , which will make the ratio  $P_{\text{exc}}/P_N = 1$  at different kinetic temperatures, yielding negative energy balance due to the excitation radiation alone, can be calculated easily. These are shown in Table IV. In practice, in order to avoid the excitation radiation,  $N^*$  has to be  $\ll N^*_{\text{critical}}$ . For a self-sustaining system of reaction no. 2,  $N^*$  critical has to be much less than  $10^{-3}$ ; and for reactions other than  $Z = 1$ , it must be still much lower.

## 7. Discussion

As the ratio  $P_{\text{rad}}/P_N \geq 1$ , a self-sustaining system is not possible.

Further, the net electrical energy gain  $G$  is obtained by

$$G = \frac{E_{\text{output}}}{E_{\text{input}}} \quad (27)$$

where

$E_{\text{output}}$  = Electrical energy gain

$$= \eta (P_N + P_{\text{rad}} + P_p)$$

$E_{\text{input}}$  = Electrical energy input

$$= \varepsilon_0^{-1} (P_p + P_{\text{rad}})$$

$\eta$  is the efficiency of converting the energy of photons, charged particles and neutrons into electrical power and is roughly equal to  $1/3$ .  $\varepsilon_0$  is the efficiency of coupling the electrical power into the gas (or system) and is  $\ll 1$ .

Thus,

$$G = \eta \varepsilon_0 \left( 1 + \frac{P_N}{P_{\text{rad}} + P_p} \right) < 1 \quad (28)$$

Even if  $\varepsilon_0 = 1$  (an ideal case),  $G$  remains less than unity showing that although the nuclear reactions can be brought about by supplying enough energy from some external source but a net energy gain is not possible. It should be mentioned here that in practice  $\varepsilon_0 \ll 1$ .

In the present consideration, the energy required for the magnetic field  $\mathbf{B}$  is neglected.

### **8. Conclusion**

A self-sustaining system as well as a net gain of energy by reactions of nuclei has been shown to be beyond the realms of realisation in a terrestrial thermonuclear plasma, irrespective of its confinement time, since the energy loss rate by radiation exceeds the energy production rate by nuclear reactions.

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**Table I.**

The quantity  $\Sigma_{(r,s)}[\langle \sigma v \rangle_{(p,q)}]_{(r,s)} Q_{(r,s)}$  is shown for a number of reactions by light nuclei at different kinetic temperatures. Therefore,  $P_N$  ( $\text{W cm}^{-3}$ ) for any density and kinetic temperature can be obtained by means of Eq. (9).

S.N.	Reaction	$\Sigma_{(r,s)} [\langle \sigma v \rangle_{(p,q)}]_{(r,s)} Q_{(r,s)} (\text{eV cm}^{-3} \text{s}^{-1})$				
		$10^3 \text{ eV}$	$10^4 \text{ eV}$	$3 \times 10^4 \text{ eV}$	$10^5 \text{ eV}$	$10^6 \text{ eV}$
1.	${}^2\text{H}({}^2\text{H}, {}^1\text{n}){}^3\text{He}$ ${}^2\text{H}({}^2\text{H}, {}^1\text{H}){}^3\text{H}$	$6.4 \times 10^{-16}$	$4.3 \times 10^{-12}$	$2.5 \times 10^{-11}$	$1.8 \times 10^{-10}$	$8.0 \times 10^{-10}$
2.	${}^3\text{H}({}^2\text{H}, {}^1\text{n}){}^4\text{He}$	$9.7 \times 10^{-14}$	$2.0 \times 10^{-9}$	$5.0 \times 10^{-9}$	$1.5 \times 10^{-8}$	$4.8 \times 10^{-9}$
3.	${}^3\text{H}({}^3\text{H}, 2{}^1\text{n}){}^4\text{He}$	$3.8 \times 10^{-15}$	$8.2 \times 10^{-12}$	$4.0 \times 10^{-11}$	$2.2 \times 10^{-10}$	$9.1 \times 10^{-10}$
4.	${}^3\text{He}({}^2\text{H}, {}^1\text{H}){}^4\text{He}$	$5.5 \times 10^{-19}$	$4.2 \times 10^{-12}$	$1.0 \times 10^{-10}$	$3.0 \times 10^{-9}$	$3.3 \times 10^{-9}$
5.	${}^3\text{He}({}^3\text{H}, {}^1\text{n}, {}^1\text{H}){}^4\text{He}$ ${}^3\text{He}({}^3\text{H}, {}^2\text{H}){}^4\text{He}$ ${}^3\text{He}({}^3\text{H}, {}^1\text{H}){}^5\text{He}$	$1.3 \times 10^{-21}$	$1.6 \times 10^{-13}$	$6.0 \times 10^{-12}$	$3.5 \times 10^{-10}$	$6.7 \times 10^{-9}$
6.	${}^6\text{Li}({}^2\text{H}, {}^1\text{n}){}^3\text{He} + {}^4\text{He}$ ${}^6\text{Li}({}^2\text{H}, {}^1\text{n}){}^7\text{Be}$ ${}^6\text{Li}({}^2\text{H}, {}^1\text{He}){}^7\text{Li}$ ${}^6\text{Li}({}^2\text{H}, {}^4\text{He}){}^4\text{He}$	$6.4 \times 10^{-23}$	$1.1 \times 10^{-13}$	$8.0 \times 10^{-12}$	$8.1 \times 10^{-10}$	---
7.	${}^7\text{Li}({}^2\text{H}, {}^1\text{n}){}^4\text{He} + {}^4\text{He}$ ${}^7\text{Li}({}^2\text{H}, {}^4\text{n}){}^5\text{He}$	$6.0 \times 10^{-23}$	$2.0 \times 10^{-14}$	$2.0 \times 10^{-12}$	$4.0 \times 10^{-10}$	---

**Table II.**

The ratio  $P_b/P_N$  as obtained by Eqs. (24a) and (24b) is shown for a number of reactions by light nuclei at different kinetic temperatures. Wherever  $P_b/P_N \geq 1$ , the bremsstrahlung only is sufficient to make the energy balance negative.

S.N.	Reaction	$P_b/P_N$				
		$10^3$ eV	$10^4$ eV	$3 \times 10^4$ eV	$10^5$ eV	$10^6$ eV
1.	${}^2\text{H}({}^2\text{H}, {}^1\text{n}){}^3\text{He}$ ${}^2\text{H}({}^2\text{H}, {}^1\text{H}){}^3\text{H}$	$1.0 \times 10^4$	4.9	1.4	$3.7 \times 10^{-1}$	$2.6 \times 10^{-1}$
2.	${}^3\text{H}({}^2\text{H}, {}^1\text{n}){}^4\text{He}$	$1.4 \times 10^2$	$2.1 \times 10^{-2}$	$1.5 \times 10^{-2}$	$9.0 \times 10^{-3}$	$8.8 \times 10^{-2}$
3.	${}^3\text{H}({}^3\text{H}, 2{}^1\text{n}){}^4\text{He}$	$1.8 \times 10^3$	2.6	$9.2 \times 10^{-1}$	$3.1 \times 10^{-1}$	$2.3 \times 10^{-1}$
4.	${}^3\text{He}({}^2\text{H}, {}^1\text{H}){}^4\text{He}$	$9.1 \times 10^7$	$3.8 \times 10$	2.75	$1.7 \times 10^{-1}$	$4.8 \times 10^{-1}$
5.	${}^3\text{He}({}^3\text{H}, {}^1\text{n}, {}^1\text{H}){}^4\text{He}$ ${}^3\text{He}({}^3\text{H}, {}^2\text{H}){}^4\text{He}$ ${}^3\text{He}({}^3\text{H}, {}^1\text{H}){}^5\text{He}$	$3.9 \times 10^{10}$	$1.0 \times 10^3$	$4.6 \times 10$	1.4	$2.4 \times 10^{-1}$
6.	${}^6\text{Li}({}^2\text{H}, {}^1\text{n}){}^3\text{He} + {}^4\text{He}$ ${}^6\text{Li}({}^2\text{H}, {}^1\text{n}){}^7\text{Be}$ ${}^6\text{Li}({}^2\text{H}, {}^1\text{He}){}^7\text{Li}$ ${}^6\text{Li}({}^2\text{H}, {}^4\text{He}){}^4\text{He}$	$2.1 \times 10^{12}$	$3.9 \times 10^3$	$9.1 \times 10$	1.7	---
7.	${}^7\text{Li}({}^2\text{H}, {}^1\text{n}){}^4\text{He} + {}^4\text{He}$ ${}^7\text{Li}({}^2\text{H}, {}^4\text{n}){}^5\text{He}$	$2.2 \times 10^{12}$	$2.2 \times 10^4$	$3.7 \times 10^2$	3.34	---

**Table III.**

The ratio  $P_c/P_N$  as obtained by Eqs. (25a) and (25b) is shown for a number of reactions by light nuclei at different kinetic temperatures and  $\beta_M = 10^{-1}$ .  $P_c/P_N \geq 1$  means the negative energy balance due to the cyclotron radiation.

S.N.	Reaction	$P_e / P_N$ for $\beta_M = 10^{-1}$			
		$10^4$ eV	$3 \times 10^4$ eV	$10^5$ eV	$10^6$ eV
1.	${}^2\text{H}({}^2\text{H}, {}^1\text{n}){}^3\text{He}$ ${}^2\text{H}({}^2\text{H}, {}^1\text{H}){}^3\text{H}$	$2.2 \times 10^2$	$3.4 \times 10^2$	$5.2 \times 10^2$	$1.2 \times 10^4$
2.	${}^3\text{H}({}^2\text{H}, {}^1\text{n}){}^4\text{He}$	$9.4 \times 10^{-1}$	3.4	$1.3 \times 10$	$3.9 \times 10^3$
3.	${}^3\text{H}({}^3\text{H}, 2{}^1\text{n}){}^4\text{He}$	$1.1 \times 10^2$	$2.1 \times 10^2$	$4.3 \times 10^2$	$1.1 \times 10^4$
4.	${}^3\text{He}({}^2\text{H}, {}^1\text{H}){}^4\text{He}$	$1.0 \times 10^3$	$3.8 \times 10^2$	$1.4 \times 10^2$	$1.3 \times 10^4$
5.	${}^3\text{He}({}^3\text{H}, {}^1\text{n}, {}^1\text{H}){}^4\text{He}$ ${}^3\text{He}({}^3\text{H}, {}^2\text{H}){}^4\text{He}$ ${}^3\text{He}({}^3\text{H}, {}^1\text{H}){}^5\text{He}$	$2.6 \times 10^4$	$6.3 \times 10^3$	$1.2 \times 10^3$	$6.3 \times 10^3$
6.	${}^6\text{Li}({}^2\text{H}, {}^1\text{n}){}^3\text{He} + {}^4\text{He}$ ${}^6\text{Li}({}^2\text{H}, {}^1\text{n}){}^7\text{Be}$ ${}^6\text{Li}({}^2\text{H}, {}^1\text{He}){}^7\text{Li}$ ${}^6\text{Li}({}^2\text{H}, {}^4\text{He}){}^4\text{He}$	$6.8 \times 10^4$	$3.9 \times 10^3$	$9.3 \times 10^2$	---
7.	${}^7\text{Li}({}^2\text{H}, {}^1\text{n}){}^4\text{He} + {}^4\text{He}$ ${}^7\text{Li}({}^2\text{H}, {}^4\text{n}){}^5\text{He}$	$3.8 \times 10^5$	$3.4 \times 10^4$	$1.9 \times 10^3$	---

**Table IV.**

The critical values,  $N^*_{\text{critical}}$ , as obtained by Eqs. (26a) and (26b) are shown for  $Z = 1$  at different kinetic temperatures, where the ratio  $P_{\text{exc}}/P_{\text{N}} = 1$  yields the negative energy balance due to the excitation radiation alone.

S.N.	Reaction	$N^*_{\text{critical}}$		
		$10^4 \text{ eV}$	$3 \times 10^4 \text{ eV}$	$10^5 \text{ eV}$
1.	${}^2\text{H}({}^2\text{H}, \text{n}){}^3\text{He}$ ${}^2\text{H}({}^2\text{H}, \text{H}){}^3\text{H}$	$2 \times 10^{-5}$	$1 \times 10^{-4}$	$1 \times 10^{-3}$
2.	${}^3\text{H}({}^2\text{H}, \text{n}){}^4\text{He}$	$5 \times 10^{-3}$	$1 \times 10^{-2}$	$3 \times 10^{-2}$
3.	${}^3\text{H}({}^3\text{H}, 2\text{n}){}^4\text{He}$	$2 \times 10^{-5}$	$1 \times 10^{-4}$	$5 \times 10^{-4}$